Chapter 5

Polarized Proton Acceleration in RHIC

5.1 Depolarizing Resonance Strengths

Without Siberian Snakes there are numerous depolarizing resonances in RHIC, both intrinsic and imperfection resonances. The strengths of the intrinsic resonances can be calculated quite accurately from the appropriate integral over the horizontal focusing fields. Fig. 5.1 shows the result for the RHIC lattice with

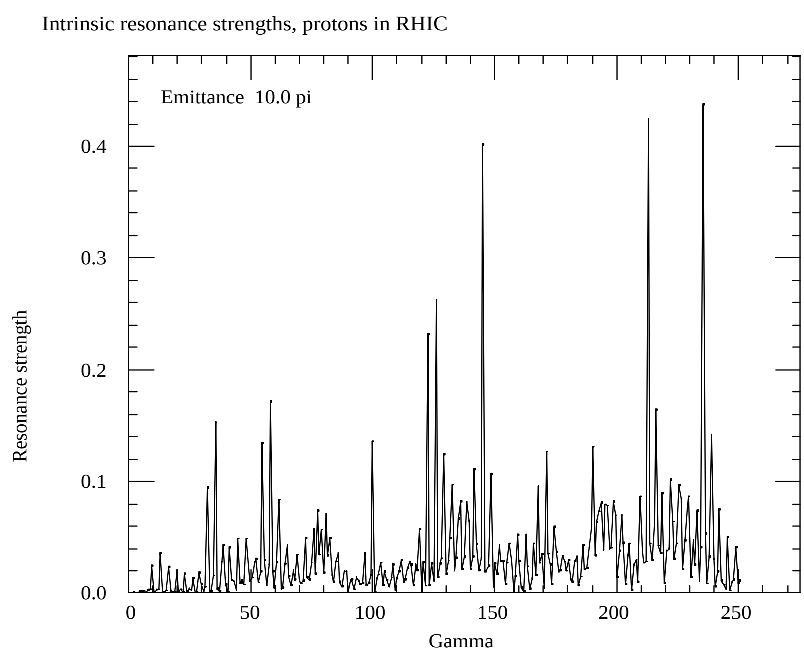


Figure 5.1: Strengths of the intrinsic depolarizing resonances in RHIC calculated for the RHIC lattice and for both $\beta^* = 10$ m and $\beta^* = 1$ m at all six intersection points. There is no noticeable difference of the calculated strengths for the two values of β^*.

$\beta^* = 10$ m at all intersections. A calculation with $\beta^* = 1$ m gave only a slightly different result. The calculation was performed for a particle with a normalized Courant-Snyder invariant of $\varepsilon_0 = 10\ \pi$ mm mrad. For a different value of the invariant the strength scales according to

$$\epsilon = \epsilon_0 \sqrt{\frac{\varepsilon}{\varepsilon_0}}$$

where ϵ_0 is the resonance strength for the invariant ε_0.

Important intrinsic spin resonances are located at

$$G\gamma = kP \pm \nu_y \approx mPM \pm \nu_B, \tag{5.1}$$

where k and m are integers, P is the superperiodicity of the accelerator, M is the number of FODO cells per superperiod, and $2\pi\nu_B = 2\pi(\nu_y - 12)$ is the accumulated phase advance of all FODO cells, which contain bending dipoles. The locations of the 3 strongest intrinsic resonances are

$$\begin{array}{llll} G\gamma = & 3\times 81 + (\nu_y - 12), & 5\times 81 - (\nu_y - 12), & 5\times 81 + (\nu_y - 12) \\ E = & \quad 136 \text{ GeV}, & \quad 203 \text{ GeV}, & \quad 221 \text{ GeV} \end{array}$$

where 81 is the product of superperiodicity, 3, and the “effective” FODO cells per superperiod, 27, which includes dispersion suppressors. The strengths of all 3 strong resonances are less than 0.5.

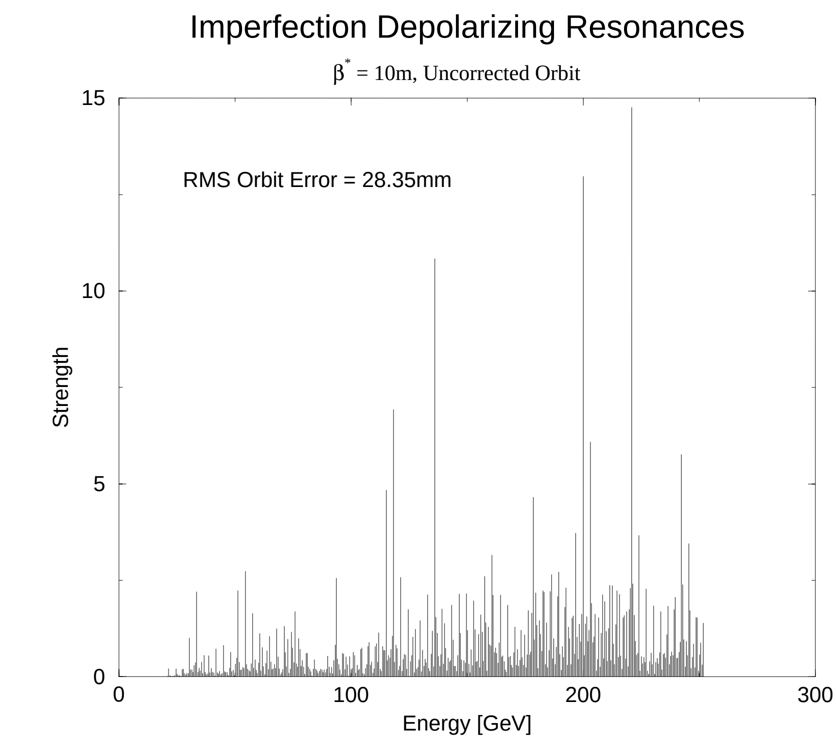
Important imperfection resonances are located at an integer closest to strong intrinsic resonances. This is clearly shown in the top part of Fig. 5.2 which shows the calculated imperfection resonance strengths for an uncorrected closed orbit obtained from a random sample of magnet misalignments with a rms spread of ± 0.5 mm, dipole roll angles with a spread of ± 1 mrad, dipole field errors of $\pm 5 \times 10^{-4}$, and position monitor errors of ± 0.5 mm. After the closed orbit correction scheme MICADO[39] was applied, the vertical closed orbit was corrected to within 0.155 mm rms. The resonance strengths are greatly reduced as shown in the lower part for Fig. 5.2. The strengths of the imperfection resonances generally increase linearly with the beam energy and are bounded by

$$\epsilon_{imp} = 0.25\,\frac{\gamma}{250}\,\sigma_y$$

where σ_y is the rms value of the residual closed orbit excursions in mm. The strength is smaller than 0.04 for all energies. Present alignment data from the RHIC CQS cold masses has shown monitor and quadrupole placement errors each well below 0.5 mm. The dipole roll angle is well below 0.5 mrad as well.[40]

5.2 Effectiveness of Siberian Snakes

With the installation of Siberian Snakes, which are local 180° spin rotators, the spin tune becomes 1/2, independent of the beam energy. Clearly the depolarizing resonance conditions cannot be met anymore as long as the fractional betatron tune $\Delta\nu_y \neq 1/2$ and therefore, in principle, no depolarization would occur. This is in fact true as long as the depolarizing resonances are not too strong. However, in the presence of



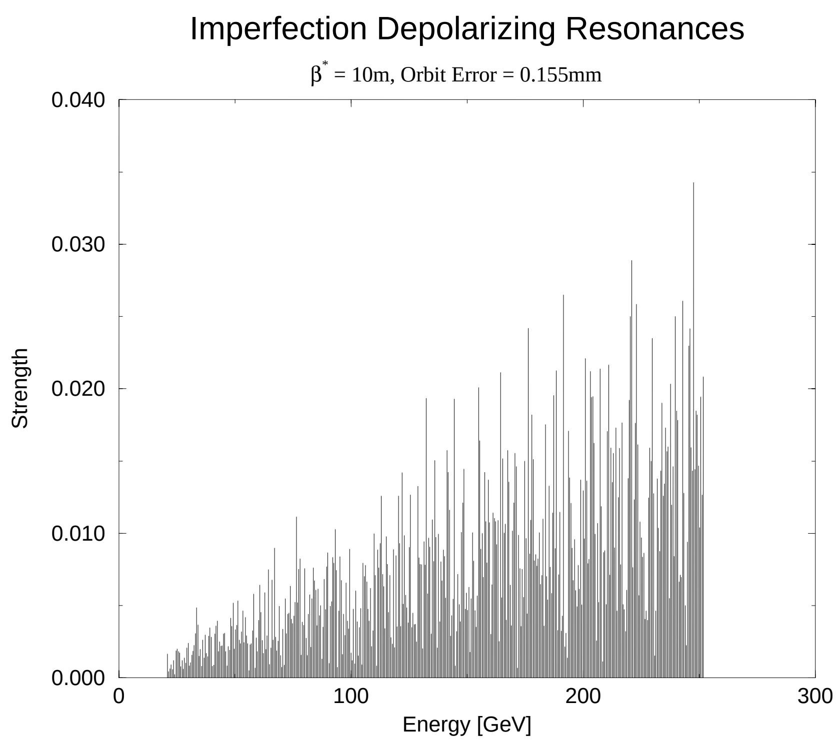


Figure 5.2: Strengths of the imperfection depolarizing resonances in RHIC calculated before and after the MICADO orbit correction scheme has been applied.

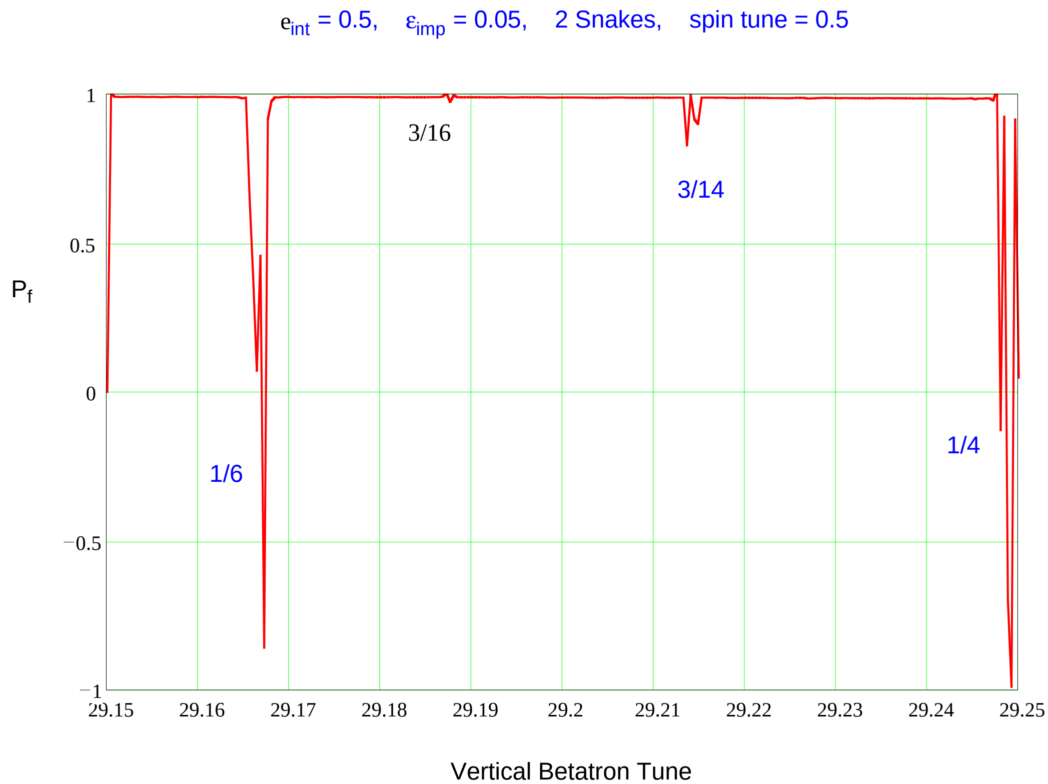


Figure 5.3: Vertical component of the polarization after acceleration through a strong intrinsic resonance and a moderate imperfection resonance shown as a function of the vertical betatron tune.

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strong resonances depolarization can occur from resonance conditions extended over more than just one turn. This leads to additional possible depolarizing resonance conditions:

$$\Delta\nu_y = \frac{\nu_{sp} \pm k}{n}$$

They are called Snake resonances[41] and n, the number of turns, is called the Snake resonance order. For two Snakes, as proposed here for RHIC, significant depolarization from Snake resonances only occurs for an intrinsic resonance strength of about 0.5 and even order Snake resonances require in addition an imperfection resonance strength of about 0.05. Fig. 5.3 shows the result of a simple 1-D spin tracking calculation through an energy region (using the RHIC acceleration rate) with an intrinsic resonance of strength 0.5 and an imperfection resonance of strength 0.05. There are clearly regions of the betatron tune that do not experience any depolarization. Since the betatron tunes of RHIC were chosen to be located between 1/6 and 1/5, the betatron tune could be placed between the Snake resonances $1/6 = 0.1667$ and $3/16 = 0.1875$. With the betatron tune including its spread located between 0.170 and 0.185, a 0.015 range, no depolarization will occur over the whole RHIC energy range up to the top energy. A more sophisticated tracking calculation, using the spin tracking code *SPINK* to track through the strongest intrinsic resonance along a corrected RHIC orbit, is found in Section 5.8.

If the betatron tune spread is too large to fit into this range, some depolarization will be caused by the Snake resonance $\Delta\nu_y = 3/16$. Tracking calculations performed with an acceleration rate of $\dot{\gamma} = 3.9$/sec showed that a Gaussian beam with $\varepsilon_{N,95\%} = 20\ \pi$ mm mrad and with 10% of the beam overlapping the 3/16 Snake resonance less than 5% of the polarization is lost for each passage through one of the strong intrinsic resonances. The final polarization after passing all 3 strong resonances would then be at least 86% of the injected polarization. It is important to note that, unlike for electron beams, the betatron tune distribution within the proton beam is basically static and does not get mixed. This means that depolarization experienced by a part of the beam is confined to this part only and will not affect the whole beam. In other words, there is no diffusion of polarization.

5.3 Sextupole Depolarizing Resonances

Spin resonances arising from sextupoles are located at

$$\nu_{sp} = kP \pm \nu_x \pm \nu_y = mPM \pm (\nu_x - 6) \pm (\nu_y - 6)$$

with resonance strength given by,

$$\epsilon_K \approx \frac{1+G\gamma}{8\pi}\sqrt{\frac{\varepsilon_x\varepsilon_y}{\pi^2}}\sqrt{\beta_x\beta_y}PM(|S_F| + |S_D|),$$

where S_F, S_D are respectively strengths of sextupoles located at the focusing and defocusing quadrupole locations. Because the emittance decreases with energy, the sextupole spin resonance strength is energy independent in hadron storage rings. For RHIC, the resonance strength is about 0.0003 at a normalized emittance of 20 π mm mrad. Such a sextupole driven resonance has been observed in the IUCF Cooler Ring.[42] However, because of their small spin resonance strength, depolarizing resonances due to sextupoles are not important as long as the betatron tunes are chosen to avoid the resonance condition, which for a spin tune of 1/2 is

$$\frac{1}{2} = kP \pm \nu_x \pm \nu_y.$$

The current working point for RHIC certainly avoids this condition.

5.4 Spin Tune Spread and Modulation

With snakes, the spin tune is independent of energy. Therefore the synchrotron motion does not give rise to spin tune spread. This has been verified indirectly in the snake experiment at the IUCF Cooler Ring, where one finds that there is no depolarization at the synchrotron side band for a 100% snake.[43] However the spin tune modulation may still arise from imperfect spin rotation in the snake, and imperfect orbital angle between snakes. The errors in orbital angle may arise from survey error, closed orbit error, and/or betatron motion.

For an imperfection resonance strength $\epsilon = 0.05$ with two snakes, the perturbed spin tune shift is given by $\Delta\nu_{sp} = \pi|\epsilon|^2/4 = 0.002$. The error in the spin rotation angle of two snakes contributes also to the imperfection resonance strength. Assuming that the relative error of the integrated field strength of each snake magnet is 10^{-3}, the error in the spin rotation angle of a snake should be about 0.5°. The effect of 2 snakes in the accelerator will give a resonance strength of the order $\epsilon_{imp}^{eq} \approx 0.004$, which is smaller than the imperfection resonances due to closed orbit errors.

The error $\Delta\theta$ in the orbital angle between the two snakes can give rise to a spin tune shift of $\Delta\nu_{sp} = G\gamma\Delta\theta/\pi$. Since the error in the orbital angle gives rise to spin tune shift and not a spin tune spread, one can compensate the effect by adjusting the spin rotation axes of the snakes. A survey error of about $\Delta\theta \approx 0.1$ mrad leads to a spin tune shift of 0.01 at the highest energy. For such a survey error, active compensation by adjusting the snake spin rotation axes is needed but is well within the tuning range of the Snake design.

The closed orbit can also cause an orbital angle error between snakes. Let us assume that the maximum closed orbit is about $\hat{a} \approx 6\sigma \approx 1.0$ mm. The angular deviation is of the order of $\Delta x'_{co} \approx \hat{a}/\sqrt{\beta\hat{\beta}}$, where $\hat{a}$ is the maximum orbit error and $\sqrt{\beta\hat{\beta}} \approx R/\nu$ is the average betatron amplitude function. The expected error is about $\Delta x'_{co} \approx 2 \times 10^{-5}$ for RHIC, which gives rise to a spin tune shift of 0.002.

Similarly a betatron oscillation can cause orbital angle modulations. The spin tune modulation is given by

$$\Delta\nu_{s,\beta} \approx \frac{1}{\pi}G\gamma\sqrt{\frac{\epsilon}{\beta}} = \frac{1}{\pi}G\sqrt{\frac{\gamma\epsilon_N}{\beta}}$$

The resulting spin tune spread is about 0.007 for a beam with 20 π mm mrad normalized emittance at 250 GeV.

Combining all the possible sources, we expect that the total spin tune spread to be about 0.009 (imperfection resonance and betatron motion) and a correctable spin tune shift of 0.012 (Snake survey error and closed orbit error).

5.5 Betatron Tune Spreads and Modulations

In avoiding snake resonances up to the 13th order, the available tune space is about 0.015. With a spin tune spread of 0.009, the betatron tune needs to be controlled to better the 0.006. The tight requirements for the spin and betatron tune spread are only relevant while accelerating through the 3 strong intrinsic resonances when the beam-beam tune shift is negligible.

At the injection energy, the space charge tune spread can be as large as 0.02 for RHIC. However, the corresponding spin resonance strength at low energy is also about a factor of 3 smaller and therefore the available tune space is much larger.

5.6 Tuning of Siberian Snakes

The desired effect of the Siberian Snakes in RHIC is to effectively rotate the spin of a proton 180° about an axis in the horizontal plane. The amount of rotation, as well as the direction of the rotation axis, through a complete helical dipole magnet each depend upon the energy of the particle, and in fact are functions of the variable

$$\kappa \equiv \frac{1+G\gamma}{B\rho} B_0$$

where B_0 is the central field of the magnet. At high energies, where $G\gamma \gg 1$ then $\kappa \approx eGB_0/mc$ which is constant. At lower energy, the Snake magnets would have to be adjusted to perform "perfect" 180° rotations.

It would be desirable to run the Snakes at constant field, thus avoiding ramped superconducting magnets and power supplies. Simulations have shown that this indeed can be done.[44] For a "perfect" Snake, the fields in the inner and outer helical dipoles would need to change by approximately 3% in accelerating from 24 GeV to 250 GeV. Suppose the Snakes were tuned to operate at one of the strongest expected intrinsic resonances in RHIC, for instance the $G\gamma = 381.82$ resonance ($\gamma \approx 212$). At lower energies, the precession angle would deviate from 180° to approximately 178° at injection. Likewise, the direction of the precession axis would be off by about 3°. Spin tracking using *Spink* has shown that lower energy intrinsic resonances can be crossed effectively with the Snake set at high energy settings. Using lower energy settings when passing through higher energy resonances is less effective, though not disastrous. Thus, the operational procedure would be to optimize the Snakes for high energy resonances, and leave them at these settings throughout the acceleration cycle.

5.7 Spin Tracking Calculations

Spin behavior in RHIC in the presence of intrinsic and imperfection spin resonances has been studied with a numerical code, *Spink* [45], that reads the lattice of the machine created by the code *Mad* [46] and makes use of 6×6 orbit matrices and 3×3 spin matrices. The input to the code is a random distribution of polarized protons in the transverse and longitudinal phase spaces, within prescribed emittance contours. The helices of Siberian snakes and rotators are also represented by matrices that describe the orbit motion as well as the spin rotation. *Spink* includes acceleration and synchrotron oscillations.

To study imperfection resonances, one creates with *Mad* an output file containing a distorted closed orbit generated by a random distribution of position and field errors in the lattice elements.

The code has been extensively checked against standard theoretical results, where applicable. For example, in the case of an isolated resonance, *Spink* produces results in good agreement with the Froissart-Stora formula [6].

Among the spin tracking studies performed so far, notable are (i) acceleration scans through the full RHIC cycle, from $G\gamma = 50$ to 500, with snakes on (Fig. 5.4), that reproduce the known resonance spectrum

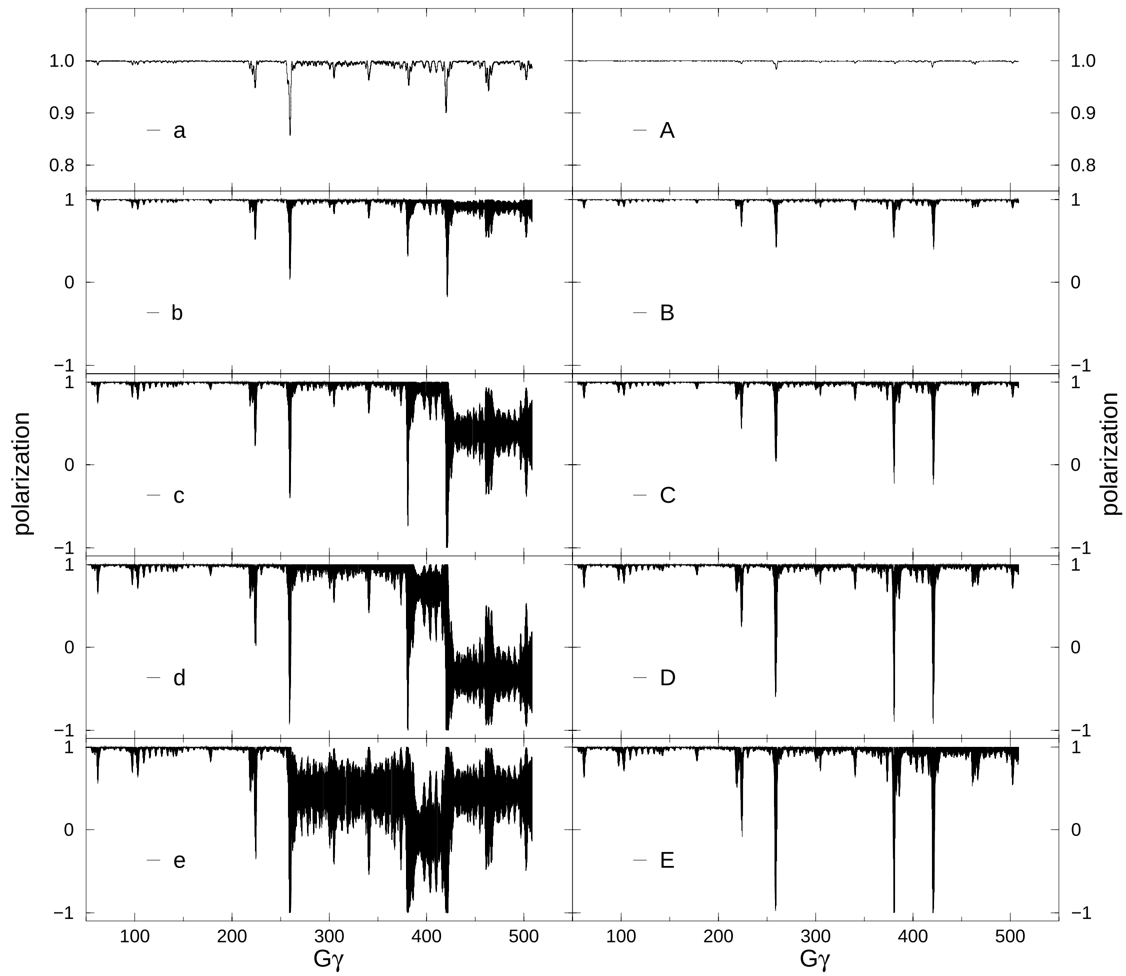

Figure 5.4: Tracking of single particle's spin in RHIC throughout the acceleration cycle. Snakes ON. Plots a - e are for vertical closed orbit deviations with rms 0.7 mm. Plots A - E are for vertical rms deviation of 0.2 mm. In each case, the horizontal orbit deviation is 0.7 mm rms. For plots a,A the particle's emittance is zero. For b,B, the particle's emittance is 5π mm-mrad. For c,C, 10π mm-mrad; d,D 15π mm-mrad; e,E, 20π mm-mrad. The RHIC tunes for this calculation are $\nu_x = 28.19$ and $\nu_y = 29.18$.

of Fig. 5.1; (ii) scan of a resonance line in storage mode (no acceleration) versus. vertical betatron tune, to find a good operating interval in tune space; (iii) test of a RF spin flipper, with rotating or oscillating magnetic field. A blow-up of a $G\gamma$ tracking interval, containing an intrinsic resonance, with snakes ON and snakes OFF is shown in Fig. 5.5.

The same code *Accsim* has also been extensively used to describe and interpret experimental results on polarized proton acceleration in the AGS [47].

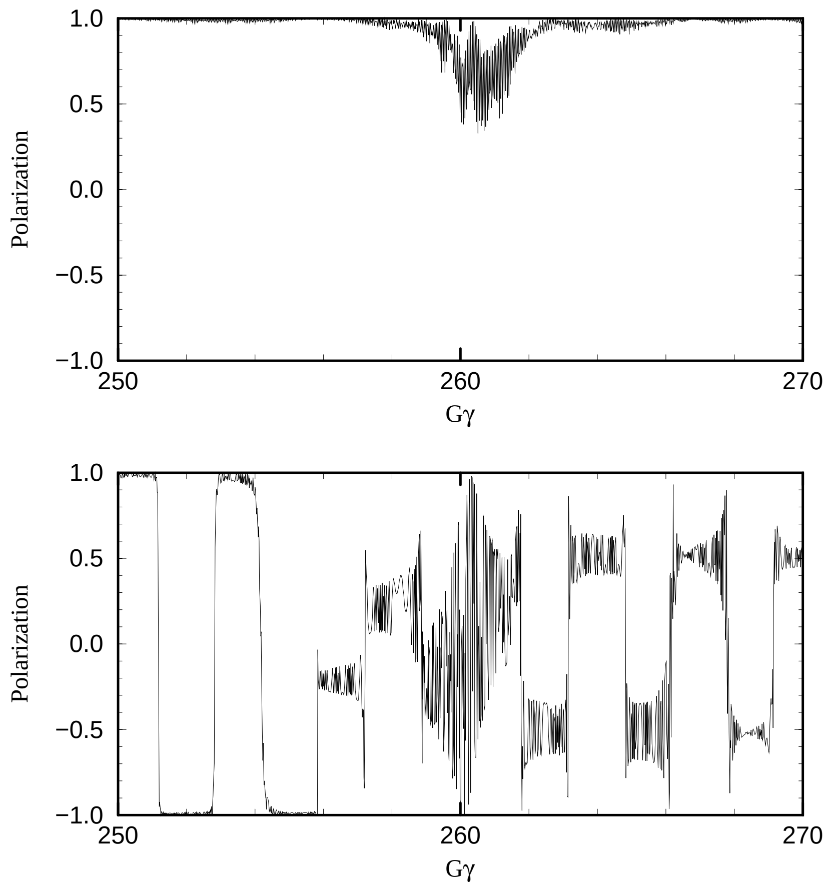

Figure 5.5: Tracking of spin through an isolated intrinsic resonance. Upper graph: snakes ON, lower graph: snakes OFF In this case, many other weak resonances show up). The tunes were again $\nu_x = 28.19$ and $\nu_y = 29.18$, and the single particle's emittance was 10π mm mrad.

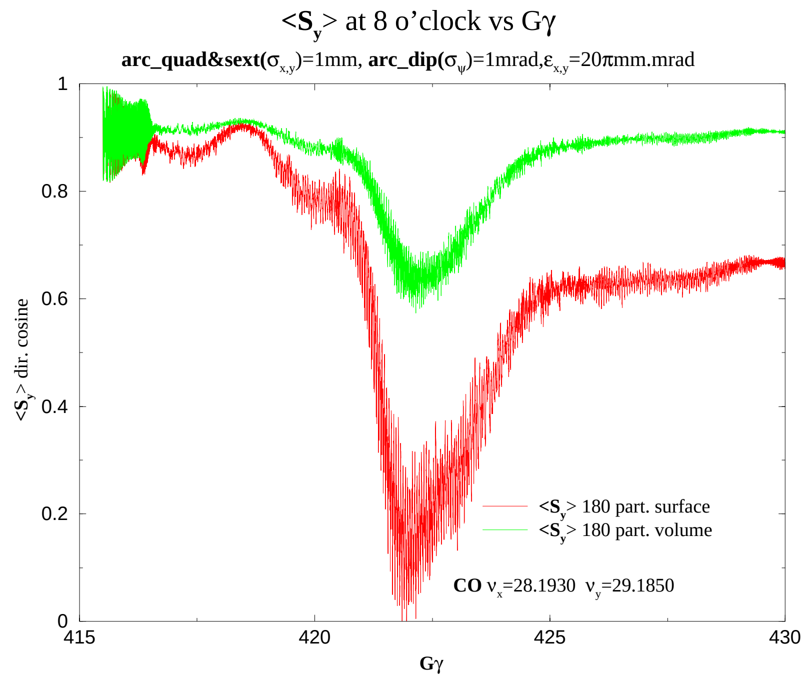

Figure 5.6: Tracking of spin through the intrinsic resonance $G\gamma = 422.18$. TOP: Average polarization of Gaussian beam (180 particles) with horizontal and vertical emittances (95%, normalized) of 20π mm-mr. BOTTOM: Average polarization of 180 particles, all with 20π emittance (horizontal and vertical).

5.8 Closed Orbit Correction Schemes

The *SPINK* computer code has been shown to reproduce the theoretically computed proton spin resonances, and their corresponding strength for an ideal RHIC machine (with magnetic elements not misaligned or having field errors). More recently, the *SPINK* code has been upgraded to allow for the study of spin resonances in the presence of random misalignments and/or field errors of magnetic elements. Two key issues can now be addressed with this version of the code. Firstly, tolerable field errors and misalignments of the magnetic elements of RHIC for which a 20π mm-mrad emittance proton beam will maintain its polarization can be explored through the various spin resonances during the acceleration cycle. Secondly, the most effective closed orbit correction scheme (MICADO, Harmonic Correction, 3-bump correction, for example) which minimizes the depolarization of the proton beam during resonance crossing can be explored.

As an example of the proton spin behavior while crossing the strongest spin resonance ($G\gamma = 422.18$) of RHIC is shown by the curves in Fig. 5.6. The top curve shows the average vertical spin component as a

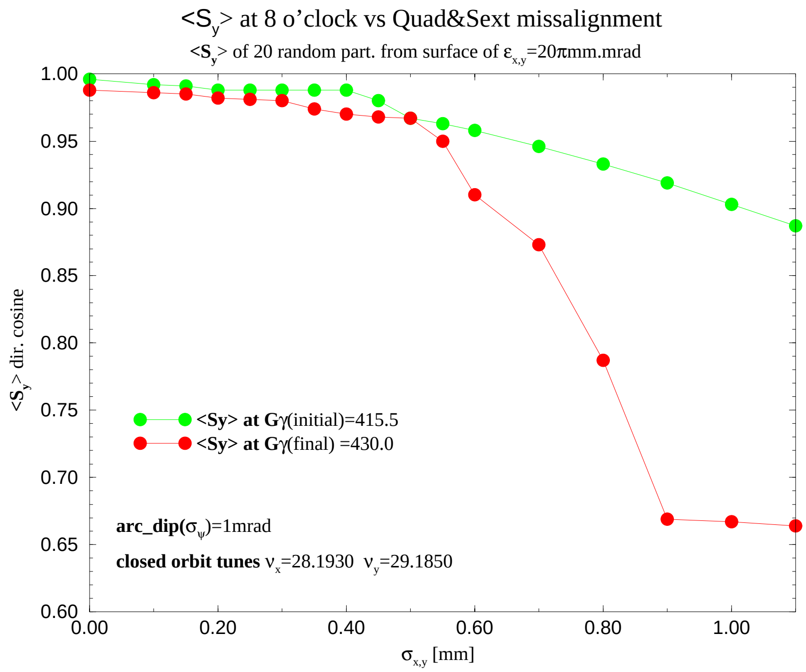

Figure 5.7: Resulting polarization through the strongest RHIC intrinsic resonance versus rms arc quadrupole and sextupole misalignment.

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function of $G\gamma$ for 180 particles randomly selected from a Gaussian beam with normalized 95% emittances $\epsilon_x = \epsilon_y = 20\pi$ mm-mr). The bottom curve is the same average but the particles are randomly selected from the surface of a beam ellipsoid with $\epsilon_x = \epsilon_y = 20\pi$ mm-mr. Both curves correspond to a RHIC machine with the same arc-quadrupole and arc-sextupole misalignments of $\sigma_{x,y} = 1.0$ mm and arc-dipole rotational misalignments of $\sigma_\psi = 1$ mr. (Only errors within ± 2.5 times the chosen rms values were allowed in the simulations discussed here.) The MICADO algorithm was applied to correct the orbit prior to tracking the particle trajectories and spins through the resonance at the nominal RHIC acceleration rate. It is worth noticing that the average spin polarization shown by the top curve well after the resonance is almost the same as the average polarization well before the resonance. A closer look at the average polarization of particles selected from the surface of the 20π beam ellipsoid (lower curve) indicates that some of these particles do not track the stable spin direction during the resonance crossing. This polarization loss decreases with the rms error of the arc-quadrupole and sextupole displacements as shown in Fig. 5.7. In this figure, the initial (at $G\gamma = 415.5$) and final (at $G\gamma = 430.0$) average vertical spin components of 20 particles (randomly selected from the surface of 20π ellipsoid) are plotted as a function of the arc-

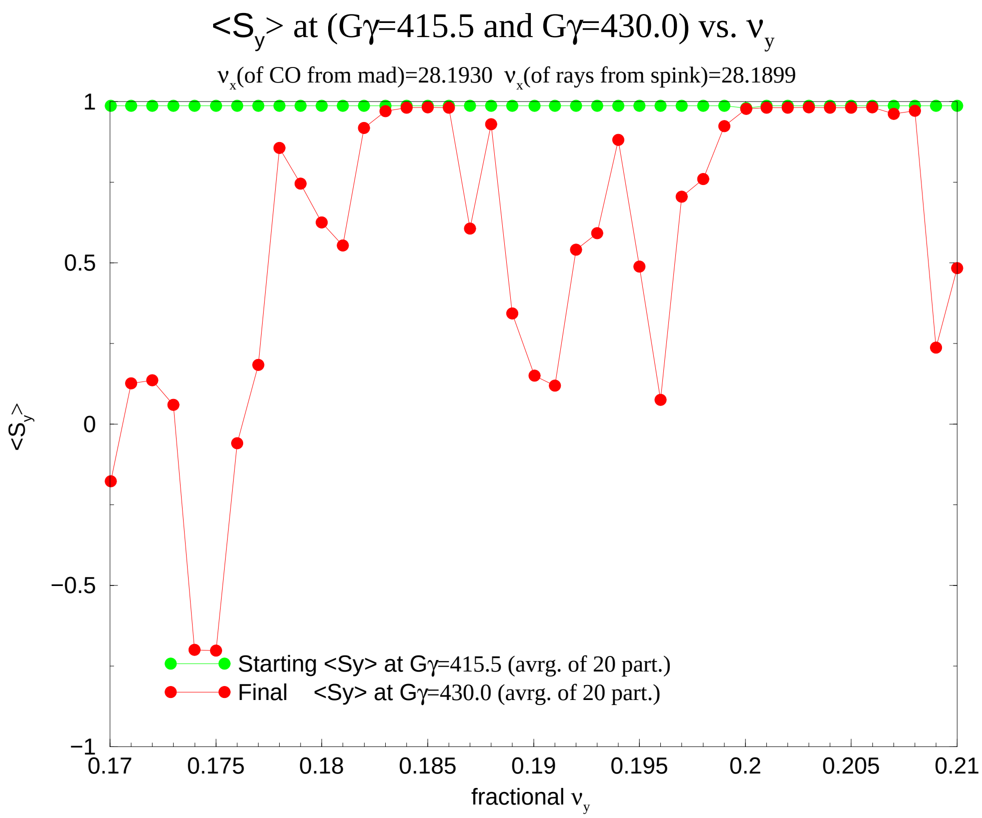

Figure 5.8: Resulting polarization through the strongest RHIC intrinsic resonance versus vertical betatron tune.

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quadrupole and arc-sextupole misalignments. The arc-dipoles maintain the same random misalignment of $\sigma_\psi = 1$ mr. The maximum and minimum spin components of the distribution are also plotted. Care was taken to ensure that both the horizontal and vertical tunes of the closed orbit were the same for each calculation.

In all the calculations the MICADO formalism minimizes the closed orbit distortion at the location of the Beam Position Monitors (BPM's) which are located just next to the arc-sextupoles and are assumed to be free of errors. Similar calculations with appropriate BPM alignment errors produce the same results as in Figs. 5.6 and 5.7.

A study of the depolarization of a proton beam crossing the $G\gamma = 422.18$ RHIC spin resonance as a function of the vertical tune is shown in Fig. 5.8. In this figure, each point represents the ratio of the final average vertical spin component to the initial average for 20 particles randomly selected from the surface of a beam ellipsoid with $\epsilon_x = \epsilon_y = 20\pi$ mm-mr. During these calculations the RHIC machine was misaligned with random arc-quadrupole and arc-sextupole rms displacements of $\sigma_{x,y} = 0.2$ mm, and arc-dipole rotations with standard deviation of σ_ψ =1 mr, and then corrected using MICADO. The horizontal tune of the closed orbit was kept at 28.1930 for each case. The first point corresponds to a vertical tune of 29.170 and the final point to 29.210 with the points in between differing by 0.001 units. There are clearly regions of betatron tune where depolarization does not occur.